Exponential derivatives

Exponential functions have their own special derivative rules. While they might look a lot like the power functions we've seen before (like x^3), remember that power functions have a variable in the base and a constant in the exponent. Exponential functions, on the other hand, have a constant in the base and a variable (or variable function) in the exponent.

If we're given an exponential function like

$$y = a^{g(x)}$$

then its derivative is

$$y' = a^{g(x)} \left(\ln a\right) \left[g'(x)\right]$$

If our exponential base is e, (if $a = e$), then the derivative rule is shown below. This should makes sense to us. The only thing different about the derivative formula below from the one above is that it's missing the $\ln a$. But of course, if $a = e$, then we get $\ln e$, which is equal to 1. And that's why the $\ln a$ disappears. So we say, given

$$y = e^{g(x)}$$

the derivative is

$$y' = e^{g(x)} \left[g'(x)\right]$$

Let's try an example with exponential base a.

Example

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Find the derivative of the exponential function.

$$y = 42^{6x}$$

Remember to use the general rule when calculating this derivative.

$$y' = 42^{6x}\left[\ln(42)\right](6)$$

$$y' = 6(42)^{6x}\ln(42)$$

Now let's try an example with an exponential base e.

Example

Find the derivative of the exponential function.

$$y = 3e^{2x}$$

Follow the general rule to take this derivative.

$$y' = 3e^{2x}(2)$$

$$y' = 6e^{2x}$$

Let's try one final, more complex example.

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Example

Find the derivative of the exponential function.

$$y = 8x^3 - \left(4^{7x}\right)\left(e^{8x+1}\right) + 6^{2x-9}$$

We have to take the derivative one term at a time, remembering to apply product rule when we get to $\left(4^{7x}\right)\left(e^{8x+1}\right)$.

$$y' = 24x^2 - \left[\left(4^{7x}\right)\left(\ln 4\right)(7)\left(e^{8x+1}\right) + \left(4^{7x}\right)\left(e^{8x+1}\right)(8)\right] + \left(6^{2x-9}\right)\left(\ln 6\right)(2)$$

$$y' = 24x^2 - \left(7e^{8x+1}\right)\left(4^{7x}\right)\left(\ln 4\right) - \left(8e^{8x+1}\right)\left(4^{7x}\right) + 2\left(6^{2x-9}\right)\left(\ln 6\right)$$

Bring that last term to the front, since we're going to leave it alone. While you're at it, pull e^{8x+1} out to the front of the other two terms.

$$y' = 24x^2 + 2\left(6^{2x-9}\right)(\ln 6) - e^{8x+1}(7)\left(4^{7x}\right)(\ln 4) - e^{8x+1}(8)\left(4^{7x}\right)$$

Group together the last two terms.

$$y' = 24x^2 + 2\left(6^{2x-9}\right)(\ln 6) - \left[e^{8x+1}(7)\left(4^{7x}\right)(\ln 4) + e^{8x+1}(8)\left(4^{7x}\right)\right]$$

Both of the last two terms contain e^{8x+1}. Factor it out.

$$y' = 24x^2 + 2\left(6^{2x-9}\right)(\ln 6) - e^{8x+1}\left[(7)\left(4^{7x}\right)(\ln 4) + (8)\left(4^{7x}\right)\right]$$

Both of the last two terms contain 4^{7x}. Factor it out.

$$y' = 24x^2 + 2\left(6^{2x-9}\right)(\ln 6) - e^{8x+1}\left(4^{7x}\right)\left[(7)(\ln 4) + (8)\right]$$

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Get rid of the parentheses you don't need.

$$y' = 24x^2 + 2\left(6^{2x-9}\right)\ln 6 - e^{8x+1}\left(4^{7x}\right)\left(7\ln 4 + 8\right)$$

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